Sum of Odd Squares

Submission deadline: February 28^{th} 2018

The Swiss mathematician Lenohard Euler in 1735 discovered that

$$1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\frac{1}{6^2}+\cdots=\frac{\pi^2}{6}$$

Can you find the value of the infinite series

$$1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\frac{1}{9^2}+\cdots?$$

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Discussion

Multiply the equation

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \cdots = \frac{\pi^2}{6} \tag{1}$$

by $1/2^2$. Then we have

$$\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \frac{1}{8^2} + \frac{1}{10^2} + \frac{1}{12^2} + \cdots = \frac{1}{2^2}\frac{\pi^2}{6} \tag{2}$$

Next, subtract the equation (2) from equation (1). This yields,

$$\begin{aligned} 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{9^2} \cdots &= \frac{\pi^2}{6} - \frac{1}{2^2}\frac{\pi^2}{6} \\ &= \frac{\pi^2}{8} \end{aligned}$$

The problem can also be solved by using Fourier series of certain functions.