An Alternating Series

Submission deadline: November 30^{th} 2018

Given that

$$1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\frac{1}{6^2}+\cdots=\frac{\pi^2}{6}$$

find

$$1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\frac{1}{5^2}-\frac{1}{6^2}+\cdots$$

The problem was solved by

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Discussion:
It is known that

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \cdots = \frac{\pi^2}{6} \tag{1}$$

Multiplying equation (1) by $1/2^2$ yields that

$$\frac{1}{2^2} + \frac{1}{(2 \cdot 2)^2} + \frac{1}{(2 \cdot 3)^2} + \frac{1}{(2 \cdot 4)^2} + \frac{1}{(2 \cdot 5)^2} + \frac{1}{(2 \cdot 6)^2} + \cdots = \frac{1}{2^2}\frac{\pi^2}{6} \tag{2}$$

Now, subtracting two times equation (2) from equation (1) results in

$$\begin{aligned} 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + \cdots &= \frac{\pi^2}{6} - \frac{1}{2}\frac{\pi^2}{6} \\ &= \frac{\pi^2}{12} \end{aligned}$$

Also see solution to February 2018 problem.