# Symmetry

**Submission deadline: August $28^{\text{th}}$ 2023**

Solve the system

$$2x^2 - 4xy + 3y^2 = 36$$
$$3x^2 - 4xy + 2y^2 = 36$$

The problem was solved by

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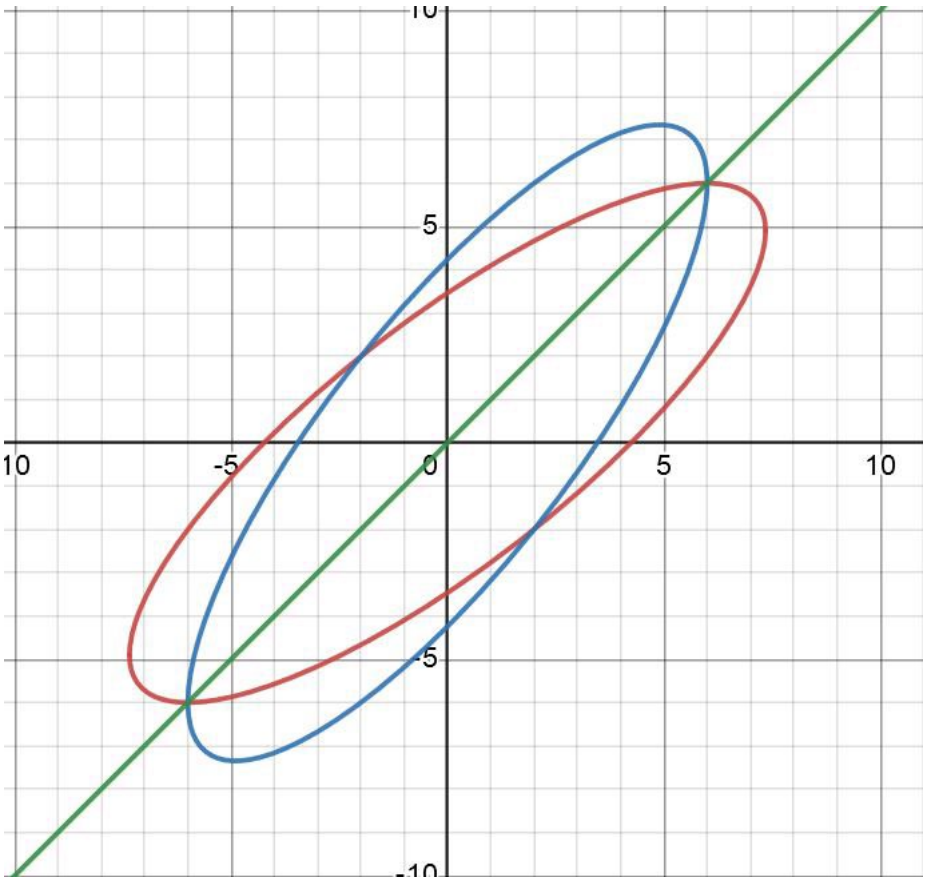
Discussion:

Taking the difference of the two equation results in $y^2 = x^2$. Hence $y = \pm x$.

Letting $y = x$ in the first equation yields $y^2 = 36$. Thus, we get solutions (6, 6) and (-6,-6).

Letting $y = -x$, in the first equation results in $y^2 = 4$. Hence we get (-2,2) and (2,-2).

Geometric interpretation: The first equation represents an ellipse $E_1$. Notice that the second equation can be obtained by switching $x$ and $y$. Thus, the second equation represents the reflection of $E_1$ on the line $x = y$. Thus, the points of intersection of $E_1$ and the line $x = y$, stay fixed and those are $(6, 6)$ and $(-6, -6)$. The points of intersection of $E_1$ and the line $y = -x$, gets interchanged and those are (2,-2) and (-2,2). See the diagram in the next page.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